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**What does a ship have to do with math?**

**Video:** <https://youtu.be/XFU1Aburjwk>

**Lesson Plan**

**Teacher Note:** Please preview the entire video and pre-work the solutions in order to anticipate students’ needs, misconceptions and materials unique to your classroom.

You will also need to determine the background knowledge of your students regarding the following topics, and decide the best method for providing that background in order to support the conceptual understanding of the mathematics shown in the video.

* + Density
	+ Displacement
	+ Buoyancy
	+ Volume
	+ Solving 1-step equations

**Common Core Mathematical Content Standards**

* 6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.
* 7.G.6 Solve real world mathematical problems involving area, volume and surface area of two and three dimensional objects.
* High School – Math Modeling
* G-MG Apply geometric concepts in modeling situations.

**Common Core Mathematical Practice Standards**

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

4. Model with mathematics

6. Attend to precision

**Company Information**

**Fincantieri Marinette Marine (FMM)** was founded in 1942 along the Menominee River in Marinette, Wisconsin to meet America's growing demand for naval construction. From humble beginnings with a contract to build five wooden barges, **FMM** has grown into a world-class shipbuilder, having designed and built more than 1,500 vessels.

Parent company, **FINCANTIERI**, has recently completed a $73.5 million capital expansion program for **Fincantieri Marinette Marine** which has transformed **FMM** into a modern shipbuilding powerhouse, now with 550,000 square feet of manufacturing, warehouse and receiving space, and the capacity to simultaneously build six Littoral Combat Ships in serial production. **FMM** employs cutting-edge computer-controlled manufacturing equipment and has heavy-lift capabilities to meet the most demanding requirement.

**Summary**

* If a ship building company builds a boat that doesn’t float, that business will not be around very long! The use of the discoveries made by the Greek scientist and mathematician, Archimedes is critical to determining a ships ability to float. In this video you will explore a real world application of Archimedes’ discoveries and the mathematics needed to apply them to navel architecture.

**Pre-Activity Discussion:**

* Archimedes Principle and other principles of fluid mechanics.
	+ Any object completely or partially submerged in a liquid is acted on by an upward (or buoyant) force.
	+ Magnitude of the buoyant force = weight of the displaced liquid
	+ Volume of liquid displaced = volume of submerged object
	+ Weight of liquid displaced = Weight of floating object
	+ An object will sink down until its weight matches the weight of the water displaced.
	+ Archimedes' principle is a law of physics fundamental to fluid mechanics.
* To decide if an object sinks or floats, compare the objects weight/mass and the upward buoyant force.
	+ If the object’s mass/weight is less than the buoyant force (weight of the displace liquid), the object will float.
	+ If the object’s mass/weight is more than the buoyant force (weight of the displaced liquid), the object will sink.
* Formula for density
	+ In science: D = m / V (density equals mass divided by volume. It is normally measured in grams per cubic centimeter).
	+ In naval architecture: ρ = Δ / V (rho equals delta divided by volume, where rho is the density of the water displaced, delta is the mass of the water displaced, and V is the volume of water being displaced. It is measured in metric tons per cubic meter)
* Density of water is 1 gram per cubic centimeter OR 1 metric ton per cubic meter.
* One mL of water weighs 1 gram and has a volume of 1 cubic cm.

**Differentiation:**

* The questions on the student handout are scaffolded to meet the needs of students who may need extra support.
* Eliminating some of the added questions and/or the table, and just posing the questions from the video would be a possible differentiation strategy for students who do not need the extra support.
* Students may also benefit by working with others as part of a partner/group investigation.

**Information that will be given in the video:**

* In naval architecture the formula for density is ρ = Δ / V (rho equals delta divided by volume, where rho is the density of the water displaced, delta is the mass of the water displaced, and V is the volume of water being displaced. It is measured in metric tons per cubic meter)
* The common science formula for density that will be used for this lesson plan will be D = m / V (density equals mass divided by volume. It is normally measured in grams per cubic centimeter)
* Ship is in the shape of a rectangular prism with length 10 meters, width 4 meters and height 4 meters
* Ship’s weight is 50 metric tons (MT)
* Ship needs to transport 5 metric tons (MT) of material.
* The ship will sink down until its weight matches the weight of the water displaced. While this occurs, the length and width remain constant, and the height/depth the ship sits in the water will increase until it reaches equilibrium

**Part 1: (0:00 – 2:12)**

* Ship is in the shape of a rectangular prism with length 10 meters, width 4 meters and height 4 meters
* Ship’s weight is 50 metric tons (MT)
* Ship needs to transport 5 metric tons (MT) of material.
* Question 1: How heavy will the ship be with all its cargo?
* Question 2: What is the ship’s volume?
* Question 3: What is the maximum displacement of the ship?
* Question 4: Will the ship sink or float?
* Before showing Part 2, have students share their solutions answers and justify their thinking.

BREAK 1

**Part 2: (2:15 – 2:48)**

* Solutions are given for the four questions posed in Part 1
* Question 5: How deep will the ship float?
	+ The ship will sink down until its weight matches the weight of the water displaced.
	+ While this occurs, the length and width remain constant, and the height/depth the ship sits in the water will increase until it reaches equilibrium.
* Before showing Part 3, have students share their answers and problem-solving methods.

BREAK 2

**Part 3: (2:51 – 3:37)**

* Discuss any errors or misconceptions in student thinking and solution.

**Extension:**

* 56 seconds into the video the engineer states “If the ship’s overall density is less than the density of the water the ship will float.” Use the measurements given in the video and the formula for density to prove the ship can float using a different method than what was shown in the video.
* Use dimensional analysis to prove the conversion for the density of water (1g/cm3 = 1MT/m3)
* Using the same cargo weight, investigate different lengths, widths and heights that would maximize volume of the ship while still allowing it to float.
* Adjust the dimensions of the ship and the weight of the cargo to values that mirror actual great lakes cargo ship dimensions and cargo weights and determine whether it will float.
* Build 3D models in various shapes and calculate volume to determine if the ship will still float.
* Research Archimedes and his contributions to mathematics and science.

**Student Handout - *What does a ship have to do with math?***  Name(s):

**Pre-Video Discussion:**  *Notes on important background information.*

**Problem:** *Will the ship float?*

 **Break 1:**

1. Information presented in the video:
2. How heavy is the ship with all its cargo?
3. What is the ship’s volume?
4. What is the maximum displacement of the ship?
5. Will the ship float? Justify your decision.

**Break 2:**

1. Additional information presented in the video:
2. How deep will the ship be in the water if it can float?

**ANSWER KEY – What does a ship have to do with math?**

1. Information presented in the video:
* **Ship is in the shape of a rectangular prism with length 10 meters, width 4 meters and height 4 meters**
* **Ship’s weight is 50 metric tons (MT)**
* **Ship needs to transport 5 metric tons (MT) of material.**
1. How heavy is the ship with all its cargo?

**Ship weight + Cargo weight = Total weight**

**50 MT + 5 MT = 55 MT**

1. What is the ship’s volume?

**Volume of a rectangular prism = area of the base x height**

**Volume of a rectangular prism = length x width x height**

**Volume of the ship = 10m x 4m x 4m**

**V = 160 m3**

1. What is the maximum displacement of the ship?
* **Maximum displacement of water would occur if the ship was totally submerged.**
* **Density of water is constant. It is 1gm per cm3 or 1MT per m3**
* **Using the formula for density:**

**Density of the water displaced = (weight/mass of the water displaced) / (volume of water displaced)**

Volume of liquid displaced equals volume of submerged object (Archimedes)

This is what we are trying to find (x)

Known value

$$\frac{1 MT}{1m^{3}}= \frac{? MT}{160 m^{3}}$$

**The maximum weight/mass of the water displaced or the maximum displacement of the ship is 160 metric tons.**

1. Will the ship float? Justify your decision.
	* + **Archimedes Principle - the magnitude of the upward, buoyant force = the weight of the displaced liquid.**
		+ **Therefore from above, the maximum magnitude of the buoyant force acting on the ship is equal to the maximum possible weight of the displaced water or 160 MT.**
		+ **The total weight of the ship and cargo is only 55 MT.**
		+ **To determine if a ship sinks or floats, compare the ship’s weight/mass and the upward buoyant force (weight of the displaced water).**
			- **If the ship’s mass/weight is less than the weight of the displaced water (buoyant force), the ship will float.**
			- **If the ship’s mass/weight is more than the weight of the displaced water (buoyant force), the ship will sink.**

**55MT < 160MT**

**The ship will float.**

**Break 2:**

1. Additional information presented in the video:
* **The ship will sink down until its weight matches the weight of the water displaced.**
* **While this occurs, the length and width remain constant, and the height/depth the ship sits in the water will increase until it reaches equilibrium.**
1. How deep will the ship be in the water if it can float?
* **We want to find how much of the ship (the rectangle) will be submerged.**
* **We know that the length and width measurements of the submerged part of the ship won’t change, but the height (depth) of the part under water is what we are interested in.**
* **If the ship sinks down until its weight matches the weight of the water displaced, then because we calculated the weight of the ship and the cargo to be 55 MT, as it is floating the weight of the water it displaces will also be 55MT.**
* **Using the formula for density:**

**Density of the water displaced = (weight/mass of the water displaced) / (volume of water displaced)**

Weight of water displaced equals weight of floating ship (Archimedes)

This is what we are trying to find (x)

Known value

$$\frac{1 MT}{1m^{3}}= \frac{55 MT}{? m^{3}}$$

**If the density of water is 1MT per cubic meters, then 55 MT of water displaced would have a volume of 55 cubic meters.**

* **Archimedes again!**

**Volume of water displaced = Volume of the submerged part of the ship**

 **55 m3 = length x width x height**

 **55 m3 = (10 meters)(4 meters)( h )**

 **55 m3 = (40h) m2**

 **55m3 / 40 m2  = h**

 **1.375 m = h**

 **The ship would float at a depth of 1.375 meters**